Interaction of Solar Wind with the Moon and Possibly Other Planetary Bodies

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A three-dimensional kinetic theory description of the interaction of solar wind with a nonmagnetic, nonconducting and absorbing planetary body is given. A perturbation scheme with a perturbation parameter β, the ratio of plasma pressure to magnetic pressure is used. Solving the zero-order equations and integrating over the phase space, the ion density distribution on the antisolar side of the planetary body is obtained. The results for large relative Larmor radius show some complicated periodic structures in the ion density distribution. The calculated perturbation in magnetic field is directly proportional to the parameter β, and does not depend on the angle between the solar wind velocity and the magnetic field. These results agree qualitatively with those found by Explorer 34 and 35.

I. Introduction

DURING the past ten years, considerable effort has been expended on understanding of the solar wind and its interaction with the planetary bodies.[1-3] Theoretical studies of the interaction of solar wind with the moon have been conducted by Michel,[4] and Johnson and Midgley,[5] using a hydromagnetic or continuous description, and by Whang,[6-8] using a free-molecular, one-dimensional guiding center approximation.

As discussed by Bernstein,[9] there are two levels of plasma description. One is based on the direct solution of the Boltzmann transport equation, and is usually referred to as the kinetic theory description.[10,11] This method has the merit of providing a complete description of the dynamics of a plasma, but it is mathematically difficult to handle. The second method is based on using a closed set of moment equations to characterize the behavior of a plasma, and it may be referred to as the hydromagnetic (or continuum) description. This method has the merit of relative mathematical simplicity but it is limited in application. It is ordinarily used when collisions dominate, so that local thermodynamic equilibrium exists, or when the phase velocity of the wave is much larger than the characteristic thermal speed in a collision-free plasma.

In the case of solar wind plasma the magnetic field is frozen in due to its high electrical conductivity.[2] When the wind encounters a planetary body, the interaction between the wind magnetic field lines and the intrinsic or induced magnetic field lines of the planetary body may become dominant. These field lines control the plasma motion somewhat as particle-particle interactions do in ordinary gas-dynamics. Using this analogy, a hydromagnetic description of the interactions becomes appropriate. For example, the interaction between the solar wind and the geomagnetosphere can be in some sense reduced to an equivalent hypersonic blunt-body problem in continuum neutral particle gasdynamics.[12] Physically, a detached bow shock wave is formed standing in front of the subsolar side of the Earth's magnetosphere. However, for the case of interaction between the solar wind and the moon, no evidence is found for a bow shock wave from lunar orbiting satellite Explorer 35.[13,14] Only the existence of a plasma shadow region behind the moon has been observed. Therefore the kinetic theory description of the interaction seems more appropriate.

Similarly, for the interaction of the solar wind with any nonmagnetic and nonconducting planetary body, when the continuum description cannot be employed, it becomes necessary to use the kinetic theory. The kinetic description takes account of the fact that the solar wind is not a continuous medium but an aggregate of individual particles.

In the following analysis, the kinetic theory is used to describe the interaction of the solar wind with the moon. In contrast to the one-dimensional guiding center approximation, a full three-dimensional motion of the charged particle is considered here. The ion density distribution and the disturbances of the magnetic field on the antisolar side of the moon are obtained.

II. Characteristics of the Problem

Typical values of solar wind particle density, temperature, and velocity, obtained from space satellite obervations, for a quiet solar condition at a distance of 1 a.u. from the sun are roughly 5 ions/cm^3, 10^5 °K, and 400 km/sec, respectively. The magnetic field is about 5γ and the spiral angle between the streaming velocity and the magnetic field is around 135°.

Physical processes in the solar wind plasma are characterized by the following set of parameters: the collisional mean free path λ, the average thermal velocity Q, and the collision frequency ν_c. From satellite data, the calculated particle-particle collisional mean free path is extremely large ($\sim$1 a.u.). Hence, for the interaction of solar wind with the moon, the particle-particle collisions are negligible.

Other parameters which characterize the electromagnetic process are plasma (or Langmuir) frequency ω, Debye length λ_D, Larmor frequency of revolution Ω, and average Larmor radius r_g. Here, the Debye length is approximately 10m and particle density n is large enough so that $n\lambda_D^3 \gg 1$. Therefore charge neutrality is a good approximation over a length scale much larger than λ_D. The average radius of the Larmor orbits of ions based on thermal velocity is of order 100 km and the average Larmor frequency is of order 0.5 rad/sec.

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The experimental results[13] obtained from Explorer 35 indicate that the moon appears to be nonmagnetic and relatively nonconducting. Hence, it may be viewed as a relatively cold dielectric sphere which absorbs impinging solar wind plasma particles and re-emits them as cold neutral particles. The interior field is formed by the superposition of the solar wind field (free-stream value) and that contribution from the magnetization current.

The stability of the disturbed region near the moon depends on the temporal behavior of small perturbations from the stationary (steady-state) distribution. If those deviations grow with time, then the region under consideration is unstable. If, however, these arbitrary disturbances damp out, then the situation is stable. The mathematics involved in the investigation of stability is extremely complicated even for a set of linearized equations. However, general trends usually can be obtained from some simplified solutions. For the instabilities associated with the deviation of the plasma particles' velocity distribution from a Maxwellian distribution, usually, the plasma will be unstable if the distribution of electrons is non-Maxwellian and nonmonotonic (i.e., the distribution function has other maxima in addition to the maximum for the velocity equal to zero). Instabilities of this kind are called two-stream or beam instabilities[15] and considerable studies have been done on this subject in the past decade.

From the result of Kitsenko and Stepanov,[16] it can be argued that if the velocity distribution of the ions is non-Maxwellian, an instability appears only when the velocity of ions (which corresponds to the ordered motion) becomes of the order of the thermal velocity of the electrons. In the case considered here, although the ion velocity distribution deviates rather strongly from Maxwellian, the maximum velocities of the nonequilibrium ions are of the same order as the streaming velocity (which is much smaller than the thermal velocity of the electrons). Therefore, an instability of the beam type is not present.

III. Governing Equations

The governing equations for a fully ionized and rarefied plasma are the Boltzmann equations for the ions and electrons.[17,18] In gaussian units, they are

$$\frac{\partial f_i}{\partial t} + \mathbf{v}\cdot\frac{\partial f_i}{\partial \mathbf{r}} + M_i\left(\mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f_i}{\partial \mathbf{v}} = \frac{\partial f_i}{\partial t}_{\text{collision}} \quad (1)$$

$$\frac{\partial f_e}{\partial t} + \mathbf{v}\cdot\frac{\partial f_e}{\partial \mathbf{r}} - \frac{e}{M_e}\left(\mathbf{E} + \frac{1}{c}\,\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f_e}{\partial \mathbf{v}} = \left(\frac{\partial f_e}{\partial t}\right)_{\text{collision}} \quad (2)$$

where f_i (or f_e) is the distribution function of ions (or electrons), in phase space as a function of position $\mathbf{r}$ and velocity $\mathbf{v}$ at time t, $(\partial f_i/\partial t)_{\text{collision}}$ represents contributions to $\partial f_i/\partial t$ due to collision (or the interparticle forces), e is the electric charge of an electron or singly-ionized ion, and c is the speed of light.

The average electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ which satisfy Maxwell's equations are called self-consistent fields; i.e., Maxwell's equations are

$$\nabla\cdot\mathbf{B} = 0,\ \nabla\cdot\mathbf{E} = 4\pi\rho \quad (3a)$$

$$\nabla\times\mathbf{B} = (1/c)\partial\mathbf{E}/\partial t + (4\pi/c)\mathbf{J},\ \nabla\times\mathbf{E} = -(1/c)\partial\mathbf{B}/\partial t \quad (3b)$$

where ρ, the average net charge density, and $\mathbf{J}$, the average current density, are defined by the following:

$$\rho(\mathbf{r},t) = \iiint_{-\infty}^{\infty} e f_i d^3\mathbf{v} - \iiint_{-\infty}^{\infty} e f_e d^3\mathbf{v} \quad (4)$$

$$\mathbf{J}(\mathbf{r},t) = \iiint_{-\infty}^{\infty} e\mathbf{v} f_i d^3\mathbf{v} - \iiint_{-\infty}^{\infty} e\mathbf{v} f_e d^3\mathbf{v} \quad \cdot (5)$$

In general this is an extremely complicated set of equations. However, various assumptions which depend on the characteristic parameters of the problem can be made to simplify the equations considerably without loss of the main features.

As is discussed in the previous section, the flow is stable. Hence, steady state is assumed, thereby eliminating the time variable. Also, because the Debye length is much smaller than the characteristic length of the problem, charge neutrality can be assumed. Thus, only the ion density distribution is considered, while the influence of the electric field on the ion motion is assumed negligible. Furthermore, because the ion collisional mean free path is much larger than the moon's characteristic dimension, particle-particle collisions are negligible, and particle-surface collisions dominate near the surface. The collision term $(\partial f_i/\partial t)_{\text{collision}}$ can be replaced by $A(r_s,v)\delta(F)$, where $F(r_s) = 0$ is the equation of the moon's surface, $\delta(F)$ is the delta function and $A(r_s,v)$ describes the interaction of particles with the moon's surface. For a perfectly absorbing surface, as is assumed here,

$$A(\mathbf{r}_s,\mathbf{v}) = \begin{cases} -(\mathbf{v}\cdot\nabla F)f & \text{for } \mathbf{v}\cdot\nabla F < 0 \\ 0 & \text{for } \mathbf{v}\cdot\nabla F \geq 0 \end{cases} \quad (6)$$

Under these assumptions, the Boltzmann equation for ions is reduced to the Vlasov equation,

$$\mathbf{v}\cdot\partial f_i/\partial\mathbf{r} + (e/M_i c)\mathbf{v}\times\mathbf{B}\cdot\partial f_i/\partial\mathbf{v} = A(\mathbf{r}_s,\mathbf{v})\delta(F) \quad (7)$$

The characteristics for this equation are the equations of motion of the ions, i.e.,

$$d\mathbf{r}/dt = \mathbf{v} \quad (8)$$

$$d\mathbf{v}/dt = (e/M_i c)\mathbf{v}\times\mathbf{B} \quad (9)$$

$$df/dt = 0 \quad (10)$$

The first two equations give the particle trajectory, and the third equation says that the distribution f is unchanged along the particle trajectory. In the present case, the solar wind carries a steady uniform magnetic field $\mathbf{B}_0$ and moves at a steady uniform velocity $\mathbf{V}$ with respect to the moon. The angle between $\mathbf{V}$ and $\mathbf{B}_0$ is α. It is convenient to nondimensionalize all the variables in Eqs. (7–10), that is, measure all the lengths, velocities and times in terms of the lunar radius R, the thermal speed of ions $Q = (2kT/M_i)^{1/2}$ and the time R/Q necessary to travel one R at ion thermal speed, respectively. The magnetic field is measured in terms of the undisturbed magnetic field B_0, and $eB_0/QM_ic = \Omega$, where Ω is the dimensionless Larmor frequency of ions.

Since these characteristic Eqs. (8–10) are coupled with Maxwell's equations [Eq. (3)] and the average current density [Eq. (5)], a perturbation scheme is used to make the mathematics tractable and thereby solve this set of equations. Assume

$$\mathbf{B} = \mathbf{B}^{(0)} + \mathbf{B}^{(1)} + \ldots \quad (11)$$

where $\mathbf{B}^{(0)}$ is the unperturbed upstream magnetic field, $|\mathbf{B}^{(0)}| = 1$ and $B^{(1)}$ is the first-order perturbation due to the presence of the moon, $|\mathbf{B}^{(1)}| \ll 1$. The justification of this assumption will be discussed later. Then, dropping the (0)'s for convenience, the zero-order equations are,

$$d\mathbf{r}/dt = \mathbf{v},\ d\mathbf{v}/dt = -\Omega\times\mathbf{v} \quad (12)$$

The coordinates chosen are moving with the moon and their origin is located at the center of the moon. The z axis is parallel to the streaming velocity V, y axis is in the V-B plane as is shown in Fig. 1. The solution of Eq. (12) is then

$$v_x = u_\parallel \sin\alpha - u_\perp \cos\varphi \cos(\Omega t - \theta_0) \quad (13a)$$

$$v_y = -u_\perp \sin(\Omega t - \theta_0) \quad (13b)$$

$$v_z + V = u_\parallel \cos\alpha + u_\perp \sin\alpha \cos(\Omega t - \theta_0) \quad (13c)$$

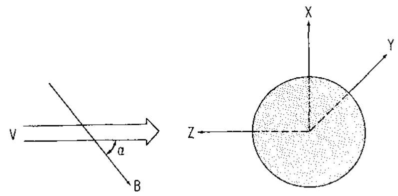

Fig. 1 The coordinate system.

$$x = x_0 + tu_{\parallel} \sin\alpha - (u_{\perp} \cos\alpha/\Omega) \times [\sin(\Omega t - \theta_0) + \sin\theta_0] \quad (14a)$$

$$y = y_0 + (u_{\perp}/\Omega)[\cos(\Omega t - \theta_0) - \cos\theta_0] \quad (14b)$$

$$z = z_0 - Vt + tu_{\parallel} \cos\alpha + (u_{\perp} \sin\alpha/\Omega) \times [\sin(\Omega t - \theta_0) + \sin\theta_0] \quad (14c)$$

where $u_{\parallel}$ is the velocity parallel to the magnetic field, and $u_{\perp}$ is the velocity perpendicular to the magnetic field. The initial conditions used to obtain the preceding solutions are

$$x = x_0, y = y_0, \text{ and } z = z_0 \text{ at } t = 0$$

and

$$v_y = 0 \text{ at } t = \theta_0/\Omega$$

By virtue of the fact that the velocity of the particle stream incident on the surface is much larger than the thermal velocity, collisions between the incident particles and the surface of the moon which is adjacent to the rarefied region behind the moon have very low probability. Therefore the specific shape of the body has little effect downstream of the moon (in the wake). Only the maximum cross section of the moon in the plane perpendicular to the incoming stream is important. Hence, in the zero-order solution the moon can be replaced by its cross section, a round disk of radius R located at the point $z = 0$. Then the ion density, n, is

$$\frac{n(x,y,z)}{n_0} = \left(\frac{1}{\pi}\right)^{3/2} \iiint_{-\infty}^{\infty} \exp(-v_x^2 - v_y^2 - v_z^2) \times H(x_0^2 + y_0^2 - 1) dv_x dv_y dv_z \quad (15)$$

where n_0 is the undisturbed ion density, and H is the Heaviside step function. To integrate the above equation, it is more convenient to use polar coordinates in velocity space, i.e.,

$$v_x = u_{\parallel} \sin\alpha - u_{\perp} \cos\varphi \cos\theta, \; v_y = u_{\perp} \sin\theta$$
$$v_z + V = u_{\parallel} \cos\alpha + u_{\perp} \sin\alpha \cos\theta \quad (16)$$

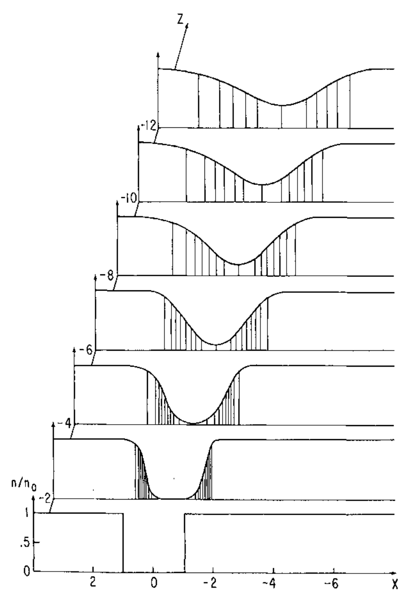

Fig. 2 Plot of ion density vs x at various distances behind the moon.

where

$$\theta = \Omega t - \theta_0$$

The Jacobian J is then equal to $u_{\perp}$, hence

$$dv_x dv_y dv_z = J du_{\perp} du_{\parallel} d\theta = u_{\perp} du_{\perp} du_{\parallel} d\theta$$

Substitution of x_0, and y_0, obtained from Eq. (14), into Eq. (15) yields

$$\frac{n(x,y,z)}{n_0} = \left(\frac{1}{\pi}\right)^{3/2} \iiint \exp(-u_{\parallel}^2 - u_{\perp}^2) \times H[g(u_{\perp},u_{\parallel},\theta)] u_{\perp} du_{\perp} du_{\parallel} d\theta \quad (17)$$

where

$$g(u_{\perp},u_{\parallel},\theta) \equiv \{x - u_{\parallel} t \sin\alpha + (u_{\perp} \cos\alpha/\Omega) \times [\sin(\Omega t + \theta) - \sin\theta]\}^2 + \{y + (u_{\perp}/\Omega) \times [\cos(\Omega t + \theta) - \cos\theta]\}^2 - 1 \quad (18)$$

and t is a parameter, defined by

$$z - Vt - u_{\parallel} t \cos\alpha - (u_{\perp}/\Omega) \times \sin\theta[\sin(\Omega t - \theta) - \sin\theta] = 0 \quad (19)$$

The variation of the magnetic field is governed by Maxwell's equations [Eq. (3)]. For a dynamo- and atmosphere-free body, such as the moon, there are three current sources resulting from the interaction of the moon with the solar wind. These three current systems[18] are denoted as Cowling, unipolar and plasma.

Sonett and Colburn[19] took account of the unipolar current system and argued that the Cowling current system was negligible. However, Blank and Sill,[20] who are interested in the magnetic field in the lunar interior, neglected the unipolar current system. They argued that because the diffusion time can be sufficiently long compared with the observed interplanetary sector structure, the Cowling current system could be significant. We will now estimate the relative importance of these three current systems. Let ω_s be the dominant frequency of the interplanetary magnetic field spectrum associated with the sector structure of the solar wind field. Assume the lunar interior consists of a low conducting outer layer and a high conducting core. From Maxwell's equations and Ohm's law, the order of magnitude of these currents may be estimated,

$$J_{\text{unipolar}} \sim (1/c)\sigma_0 VB, \; J_{\text{Cowling}} \sim (1/c)\sigma_c R\omega_s B$$
$$J_{\text{plasma}} \sim (\beta/8\pi)B/R \quad (20)$$

where σ_0 and σ_c are the conductivities of the moon's outer layer and core respectively, and β, which is of order one, is the ratio of plasma pressure to magnetic pressure. The ratio of these three currents can be written in the form,

$$J_{\text{unipolar}}:J_{\text{Cowling}}:J_{\text{plasma}} \sim \sigma_0 V/\omega_s : \sigma_c R : \beta/8\pi c/R\omega_s$$

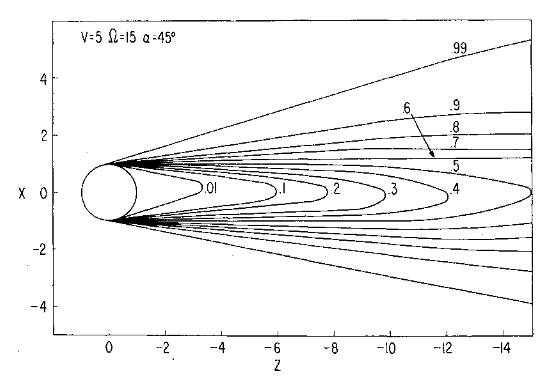

Fig. 3 Curves of constant ion density in the plane $y = 0$.

Assume that $\omega_s \sim 10^{-6}$ sec^{-1}, $\sigma_0 \sim 10^{-8}$ (Ohm-M)$^{-1}$, $\sigma_c \sim 10^{-1}$ (Ohm-M)$^{-1}$ and $R \sim 1800$ km. Substitution of these values into Eq. (20) shows that both J_{unipolar} and J_{Cowling} are much smaller than J_{plasma}. Furthermore, the magnetic field generated by the current in the lunar interior decays rapidly in a few lunar radii in the wake. Hence, in the region of present interest, the magnetic field in the lunar wake, both J_{unipolar} and J_{Cowling} are negligible. Of course, for a higher order accuracy solution, all these current systems should be included.

The plasma current is equal to the sum of magnetization current J_M and the gradient drift current J_G.† As is shown by Chandraskhar,[21] both of these currents can be expressed in terms of the total transverse energy $W_\perp$, i.e.,

$$\mathbf{J}_M = -\nabla \times (W_\perp/B)\mathbf{n},\ \mathbf{J}_G = (W_\perp/B^2)\mathbf{n} \times \nabla \mathbf{B}$$

where

$$n = \frac{\mathbf{B}}{|\mathbf{B}|},\ \text{and}\ W_\perp = \iiint_{-\infty}^{\infty} \frac{1}{2} M_i \cdot u_\perp^2 f d^3\mathbf{v}$$

Cross multiply Eq. (3) by $\mathbf{B}$ to get

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = (4\pi/c)\mathbf{J} \times \mathbf{B} \tag{21}$$

Substitute $\mathbf{J} = \mathbf{J}_M + \mathbf{J}_G$ into Eq. (21) and use the relation $\nabla \cdot \mathbf{B} = 0$ to give

$$\nabla(B^2/8\pi + W_\perp) = (1/4\pi)(\mathbf{B}\cdot\nabla)\mathbf{B} \tag{22}$$

It is noted that Eq. (22) can also be obtained by considering the momentum balance across the magnetic field.[17] In the case when $\mathbf{B}$ is not changed along its own direction, i.e., $(\mathbf{B}\cdot\nabla)\mathbf{B} = 0$, then plasma pressure balances with the magnetic pressure. However, in the case considered here the term $(\mathbf{B}\cdot\nabla)\mathbf{B}$ is not zero. To simplify Eq. (22), we use new coordinates $(\tilde{x},\tilde{y},\tilde{z})$ and unit vectors $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ where $\mathbf{e}_3$ is parallel to $\mathbf{B}^{(0)}$, $\mathbf{e}_2$ is parallel to $\mathbf{B}^{(0)} \times \mathbf{V}$, $\mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_3$, and

$$\tilde{z} = z \cos\alpha - x \sin\alpha,\ \tilde{x} = z \sin\alpha + x \cos\alpha,\ \tilde{y} = -y \tag{23}$$

In this new coordinate, Eq. (11) becomes

$$\mathbf{B} = (1 + b_3)\mathbf{e}_3 + b_2\mathbf{e}_2 + b_1\mathbf{e}_1 + \ldots \tag{24}$$

where

$$b_3\mathbf{e}_3 + b_2\mathbf{e}_2 + b_1\mathbf{e}_1 \equiv \mathbf{B}^{(1)}$$

Then by Eqs. (22) and $\nabla\cdot\mathbf{B} = 0$, we obtain

$$\nabla^2[b_3 + (\beta/2)W_\perp/W_\perp{}^0] = (\partial^2/\partial z^2)[(\beta/2)W_\perp/W_\perp{}^0] \tag{25}$$

where $W_\perp{}^0$ is the upstream total transverse momentum and β is the ratio $W_\perp{}^0/B_0{}^2/8\pi$. Here β and $W_\perp{}^0$ are known constants and $W_\perp$ is a known function of $\tilde{x},\tilde{y},\tilde{z}$, obtained from

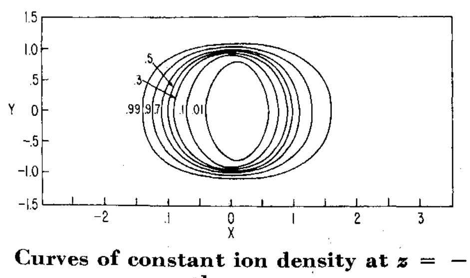

Fig. 4 Curves of constant ion density at $z = -2$ behind the moon.

† It is noted that in a recent guiding-center approximation, Whang[8] demonstrated that in the lunar wake the acceleration drift current (polarization current) is at least as large as the other currents. However, it can be shown that the acceleration drift current is zero in the present order of approximation.

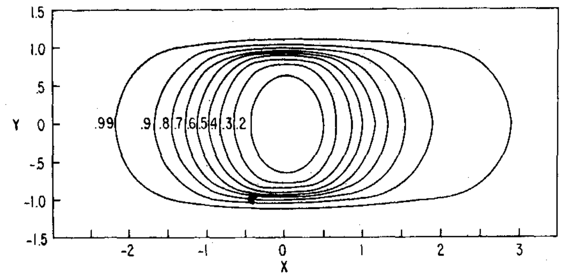

Fig. 5 Curves of constant ion density at $z = -6$ behind the moon.

previous calculations. Hence the solution of Eq. (25) is

$$b_3(\tilde{x},\tilde{y},\tilde{z}) = \frac{\beta}{2}\iiint_{-\infty}^{\infty}\left(\frac{\partial^2}{\partial z^2}\frac{W_\perp}{W_\perp{}^0}\right)\frac{1}{|\mathbf{r}-\tilde{\mathbf{r}}|}d^3\mathbf{r} - \frac{\beta}{2}\frac{W_\perp}{W_\perp{}^0} \tag{26}$$

and

$$b_1(\tilde{x},\tilde{y},\tilde{z}) = \int (\partial/\partial x)[b_3 + (\beta/2)W_\perp/W_\perp{}^0]dz + c_1 \tag{27}$$

$$b_2(\tilde{x},\tilde{y},\tilde{z}) = \int (\partial/\partial y)[b_3 + (\beta/2)W_\perp/W_\perp{}^0]dz + c_2 \tag{28}$$

where c_1 and c_2 can be determined from the boundary conditions that at upstream b_1 and b_2 are both zero. Once having obtained $W_\perp$, we integrate the preceding equations to obtain the first-order perturbation in magnetic field.

It can be shown that both the integrals and the ratio $W_\perp/W_\perp{}^0$ in these equations are less than one. And the perturbation in magnetic field, $\mathbf{B}^{(1)}$, is directly proportional to β. Taking β as the perturbation quantity, the asymptotic expansion obtained approaches the exact solution when β approaches zero. Hence the perturbation scheme used here is of rational approximation,[23] and is valid when $|\mathbf{B}^{(1)}|$ is much less than one. In the next section, some numerical results and more discussion are given.

IV. Results and Discussion

The first-order solution obtained shows that as the solar wind flows past the moon, the ion density and magnetic field are disturbed mainly downstream from the moon. A typical plot of the ion density vs x at various distances z behind the moon is shown in Fig. 2. An empty cavity or void region is produced in the near-downstream vicinity of the moon. Around the void region is a transition region, where the ion density changes from void condition to the undisturbed

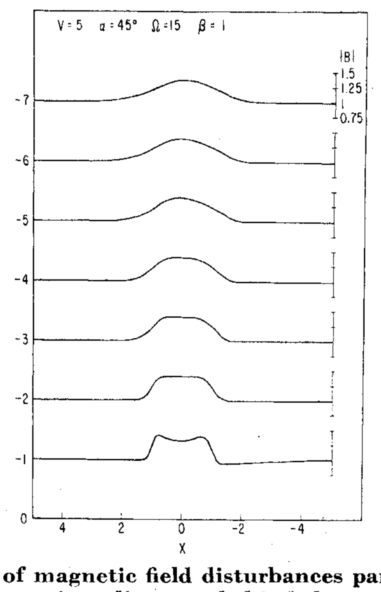

Fig. 6 Plot of magnetic field disturbances parallel to B_0 vs x at various distances behind the moon.

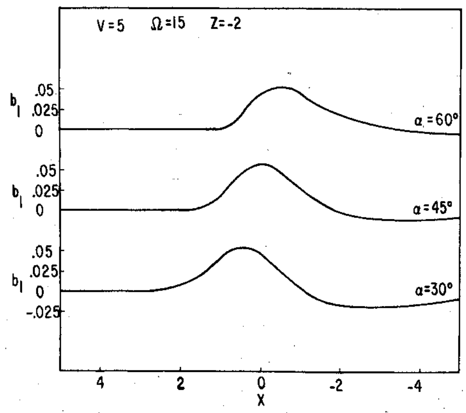

Fig. 7 Plot of magnetic field disturbances parallel to e_1 vs x at $z = -2$ behind the moon for various α.

condition. Hence, the wake may be defined as consisting of a void region (or plasma umbra in Ref. 22), where $n < 0.01$ and a transition region (or plasma penumbra in Ref. 22), where $0.01 \leq n \leq 0.99$. The constant ion density contours in the plane $y = 0$ are plotted in Fig. 3. The disturbed flow forms a long and nonaxially symmetrical wake on the anti-solar side of the moon.

The wake is not isotropic if the flow direction does not coincide with the magnetic field direction. The wake expends outward more along the x direction than along the y direction as the distance behind the moon increases. This is clearly shown in Figs. 4 and 5, which are constant ion

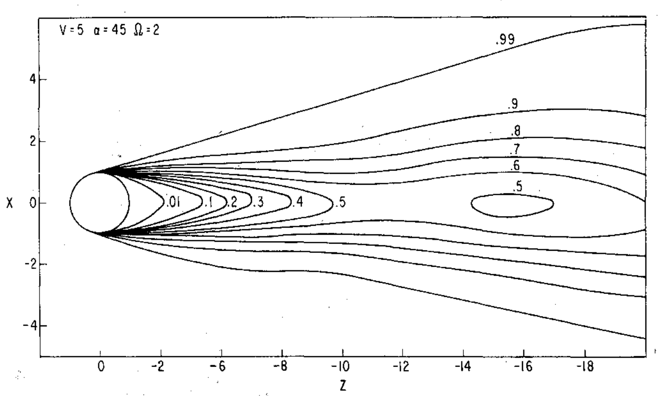

Fig. 8 Curves of constant ion density in the plane $y = 0$ for Ω equals to 2.

density curves in the xy plane behind the moon at $z = 2$ and 6.

The perturbation of the magnetic field in the $\mathbf{B}_0$ direction exhibits a very complex nature, as is shown in Fig. 6. Across the wake, the magnetic field varies above and below the ambient value in the transition region, while in the void region it stays above the ambient value and reaches a maximum.

The disturbances of magnetic field perpendicular to that of $\mathbf{B}_0$ are mainly in the $\mathbf{e}_1$ direction. The distribution of these disturbances b_1 are plotted in Fig. 7. In general, the location

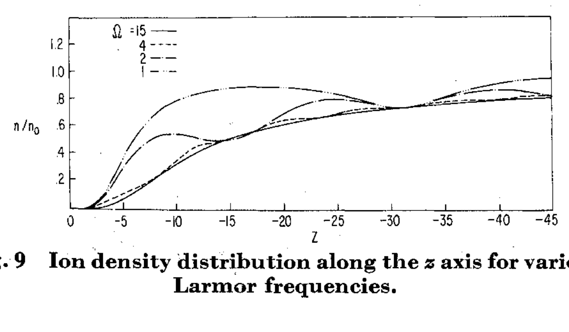

Fig. 9 Ion density distribution along the z axis for various Larmor frequencies.

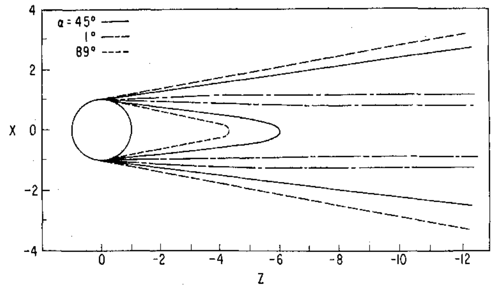

Fig. 10 Constant ion density curves of $n = 0.1$ and $n = 0.9$ for various angles.

of the peaks shift to large x as α decreases. The amplitude of the peaks are relatively independent of α and they decay in proportion to the inverse square of distance from the body. It is also noted that, although the main magnetic disturbance is in the wake region, some small magnetic disturbances seem to persist downstream along the magnetic field direction. Most of these features, i.e., the void and transition region and corresponding magnetic field disturbances behind the moon, qualitatively agree with the observations obtained from Explorer 35.[11,12] Since limited data and understanding about the solar wind are available, no detailed matching of the calculations with the experimental observation is attempted; instead, only the general behavior of the interaction will be discussed. This might be applied to the interaction of solar wind with other planetary bodies.

In general, the wake is characterized by the parameters Ω,α,V,Γ, and β. The ion density distribution is slightly affected by Γ and β while Ω,α, and V play a major role. For small Ω (of order one), the ion density distribution in the wake has a rather complicated form (Fig. 8). Because of the gyration of the ions, the ion density varies strongly over the length of $2\pi V/\Omega$. To demonstrate this, a plot of ion density distribution along the z axis for various Ω is shown in Fig. 9. The periodic structure predominates in the near wake and is graduately smeared out by the thermal motion of the ions as z increases. Both the period and amplitude of the periodic structure are proportional to the inverse of Ω. Hence for large Ω, the periodic structure becomes small and can be neglected. Neglecting the small variations, simple averaged ion density distribution contours can be obtained, such as Fig. 4.

For very small α, if Ω is large, the wake consists of a semi-infinite cylindrical void of radius R and a thin transition layer of thickness $1/\Omega$ around the void (Fig. 10). If Ω is small, there is no void, and as mentioned before, the ion density in the wake varies strongly over the lengths $2\pi V/\Omega$. As α increases, the length of the wake or void region decreases. The average disturbance of the ion density decreases with increasing distance z from the moon in proportion to $1/(z \sin\alpha)$, as is shown in Figs. 11 and 12.

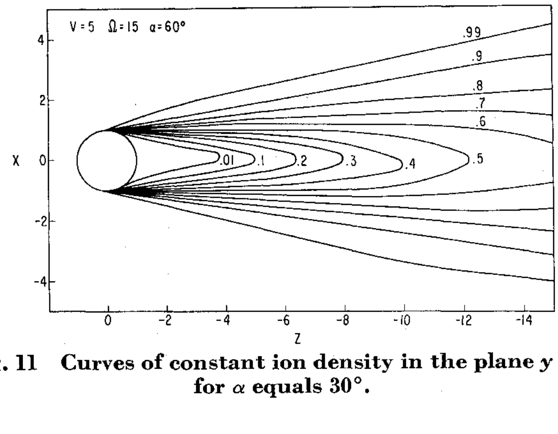

Fig. 11 Curves of constant ion density in the plane $y = 0$ for α equals 30°.

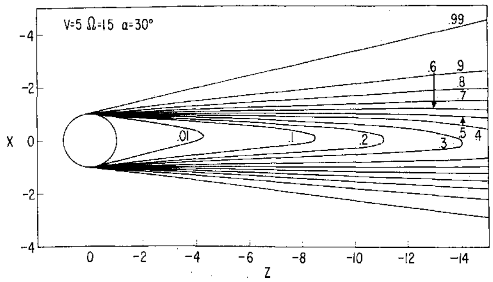

Fig. 12 Curves of constant ion density in the plane $y = 0$ for α equals 60°.

As for the effect of V, it is seen that roughly the ion density distribution in the wake is a function of V/Z. Hence changing V means stretching the length scale in the z direction. In other words, the wake becomes narrower and longer as the velocity increases (Fig. 13).

In comparing the result with the guiding center model,[6] it is seen that the general agreement is good for $\Omega = 15$ or larger. However, the wake in the present model is somewhat longer than the one calculated from the guiding center model and the maximum amplitude of the perturbed magnetic field is not changed with the angle α, in contrast to the result, strongly depend on α, obtained by Whang.[7] For smaller Ω, however, the structure of the wake becomes very complex and the guiding center model is no longer valid. Typical periodic and nonsymmetric features are easily seen in Figs. 8 and 9, where some ion density "islands" exist in the wake. It is interesting to note that similar islands have occasionally been observed by the lunar orbiter satellite.[13] Other cases where Ω might be small are: energetic particles shadowed by the moon, artificial Earth satellite, Mars satellite, asteroids, etc. In these cases, a full kinetic description of the interaction might be more appropriate.

V. Conclusion

Under the assumptions that the collisional mean free path of the solar wind plasma is much larger than the body dimension, the body is nonmagnetic and nonconducting, and the streaming velocity is much larger than the thermal velocity of ions and much smaller than the thermal velocity of electrons, the present model illustrates the essential features of the interaction of solar wind with the moon or other planetary bodies.

Because the full three-dimensional thermal motions of ions are considered, the relative Larmor radius is not limited to be small and the balance of transverse pressure is taken into account. Removing these restrictions enhances the application of this model to a wider class of interactions, where the relative Larmor radius may not be small.

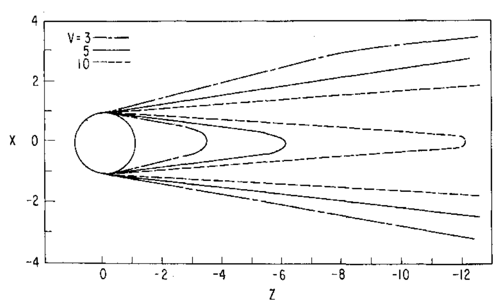

Fig. 13 Constant ion density curves of $n = 0.1$ and $n = 0.9$ for various velocities.

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